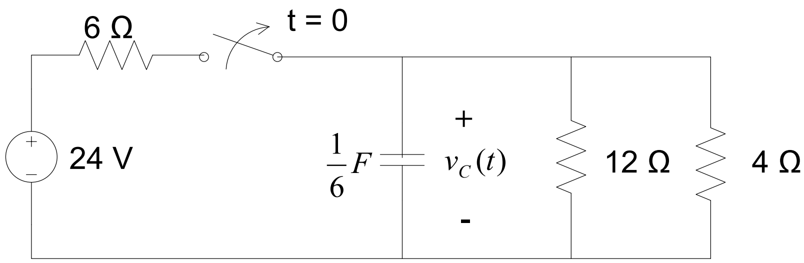
**TOPIC**

Electricity and Magnetism – Section XI – Question 14

**QUESTION**

The expression for the capacitor voltage $v_C(t)$ for $t > 0$ is




(A) 24.
(B) $24e^{-2t}$.
(C) $8e^{-2t}$.
(D) $24e^{-3t}$.

**HINT**

What makes this a transient problem and differentiates it from all the other problems we have seen so far involving inductors and capacitors, is the switch. How does a switch create a transient response for inductors and capacitors? Imagine a calm lake where the water is stand still. Then you throw a stone in the calm water and some waves are formed. But, the waves will not last forever. After sometime they will die out. Think of a switch as the equivalent of a stone and an electrical circuit with certain amount of inductance and capacitance as the equivalent of the lake. We know that inductors and capacitors for DC signals act like short and open circuits, respectively. They are calm, no waves, or transients. However, when a switch closes or opens then a change is forced on the circuit. This change will cause some waves but eventually everything will go back to steady state (calm).

To keep things simple, for an inductor solve for the current through the inductor first and then if the voltage is required then differentiate the current. Why? Because

$i_L(0^+) = i_L(0^-)$ but

$v_L(0^+) \neq v_L(0^-)$.

$$i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{\frac{-tR_{eq}}{L}}$$

then

$$v_L(t) = L\frac{di_L}{dt}$$

Note that these equations are for $t > 0$. Never substitute negative time because then you end up with a positive exponential.

For a capacitor solve for the voltage drop across the capacitor first and then if the current is required then differentiate the voltage. Why? Because

$v_C(0^+) = v_C(0^-)$ but $i_C(0^+) \neq i_C(0^-)$.

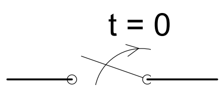
$$v_C(t) = v_C(\infty) + [v_C(0^+) - v_C(\infty)]e^{\frac{-t}{CR_{eq}}}$$

then

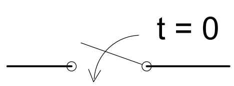
$$i_C(t) = C\frac{dv_C}{dt}$$

Note that these equations are for $t > 0$. Never substitute negative time because then you end up with a positive exponential.

Reading and understanding the switch is imperative for the correct solution of a transient problem. The movement of the switch is denoted by an arrow.




This switch for example shows that at $t$=0 it moves up (North). This concludes the following:
$t < 0$: The switch was closed.
$t < 0$: The switch opens.




This switch for example shows that at $t$=0 it moves down (South). This concludes the following:
$t < 0$: The switch was open.
$t < 0$: The switch closes.

Three Step Process for Solving a Transient Problem
Step 1: at $t < 0$: Initial Conditions
We know that everything was in steady state (Calm). Hence, the inductor will be a short circuit and the capacitor will be an open circuit. Therefore, find the current through the piece of wire (inductor) and the voltage drop across the open circuit (capacitor).
Why are we looking at $t$<0 if the above equations require $0^+$? Because we want to keep things simple and $i_L(0^+) = i_L(0^-)$ and $v_C(0^+) = v_C(0^-)$.

Step 2: at $t = \infty$: Forced Response
We know that the disturbance will go back to steady state (die out). Hence, the inductor will be a short circuit and the capacitor will be an open circuit. Therefore, find the current through the piece of wire (inductor) and the voltage drop across the open circuit (capacitor).
Why $t = \infty$ ? Time infinite is relative indicating long enough time for the transient to die out.

Step 3: at $t > 0$: Find Equivalent Resistance
Eliminate all independent sources
Current Sources, 0A implies Open Circuit
Voltage Sources, 0V implies Short Circuit

Then across the terminals where the inductor or capacitor is connected what is the equivalent resistance as seen by the inductor or capacitor?

## CONTRIBUTOR

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