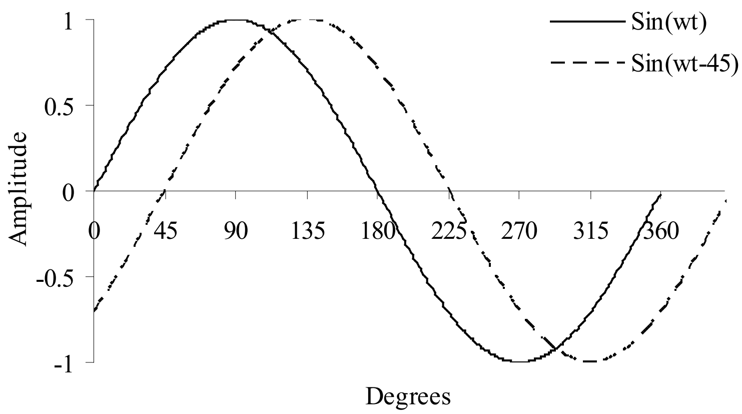
TOPIC
Electricity and Magnetism - Section XI – Question 16

QUESTION
Which statement is true?



(A) sin(wt) leads sin(wt–45°) by 45 degrees.
(B) sin(wt) lags sin(wt–45°) by 45 degrees.
(C) sin(wt) leads sin(wt–45°) by 90 degrees.
(D) sin(wt) lags sin(wt–45°) by 90 degrees.

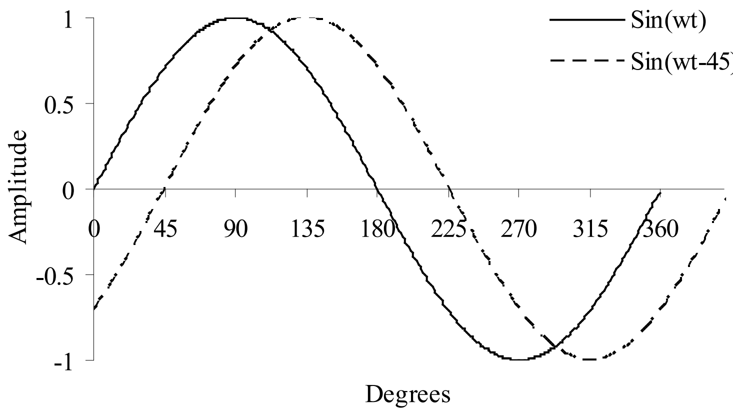
HINT
There are three ways (forms) to represent signals; Trigonometric, Phasor and Rectangular.

Trigonometric:	A $\sin(wt + \theta°)$
Phasor (Polar):	$A\angle\theta°$
Rectangular:	A $\cos(\theta) + j\sin(\theta)$

Note: In Electrical Engineering, i represents current. So, the imaginary part [$i=\sqrt{-1}$] is represented with a j. Hence, $j = i=\sqrt{-1}$.

Given the waveforms in the trigonometric form (time domain), how do we determine which waveform is leading?
The leading waveform is the one that peaks first. Hence, sin(wt) leads sin(wt–45).

How to determine the phase shift between the two signals?
Phase shift is the angle difference at two common points. For example, sin(wt)=0 at 0° whereas sin(wt)=0 at 45°. Hence, it can be said that sin(wt) leads sin(wt–45) by 45°.



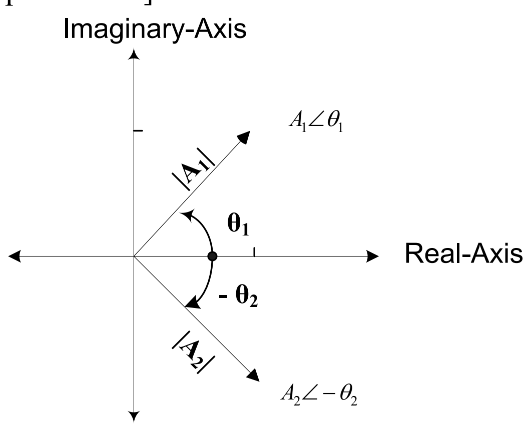
How do we write these two waveforms in polar form?

The leading waveform will have a reference angle 0° whereas the lagging waveform will have a negative phase shift. Hence, sin(wt) → $1\angle 0°$whereas sin(wt–45) → $1\angle -45°$.

Converting from Trigonometric (Time) to Polar form (Frequency).

Trigonometric waveform: A sin(wt + θ°)

Polar Form: $A\angle\theta°$ [In the frequency domain we compare signals of equal frequencies. Therefore, since the frequency is the same then the only values for comparison are the amplitude and the phase shift]



Note: You have to be cautious when comparing sine and cosine waveforms. In the frequency domain they both look the same but in reality they have a phase shift of 90°. So, be consistent, either convert to sine or both to cosine.

Incorrect: A sin(wt + θ°) →$A\angle\theta°$ and A cos(wt + θ°) →$A\angle\theta°$

Correct: A sin(wt + θ°) →$A\angle\theta°$

A cos(wt + θ°) = A sin(wt + θ°+90°) →$A\angle(\theta° + 90°)$

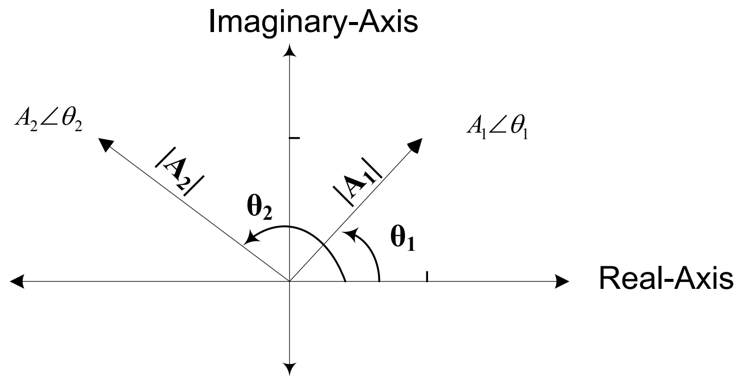
Correct: A sin(wt + θ°) = A cos(wt + θ° - 90°)→$A\angle(\theta° - 90°)$

A cos(wt + θ°) →$A\angle\theta°$

How to determine which waveform is leading?

Think of it as a "counter-clockwise race". Hence, $A_2\angle\theta_2°$ leads $A_1\angle\theta_1°$.

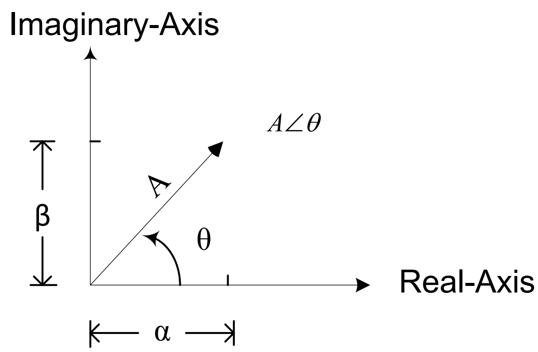
The phase shift between the two waveforms is θ_2–θ_1.



Converting from Polar to Rectangular.

Polar Form: → $A\angle\theta°$
Rectangular:→ α ± jβ

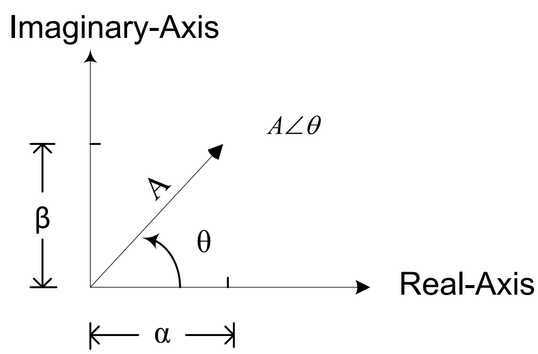
α = Acos(θ)
β = Asin(θ)



Converting from Rectangular to Polar.

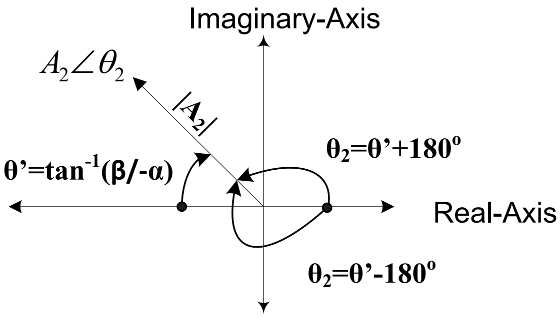
Rectangular:→ α ± jβ
Polar Form: → $A\angle\theta°$

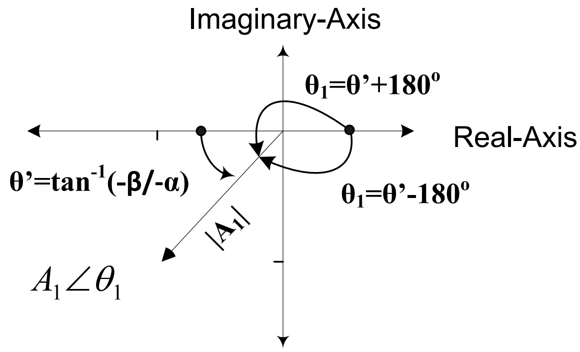
$$A = \sqrt{\alpha^2 + \beta^2}$$
$$\theta = tan^{-1}\left(\frac{\beta}{\alpha}\right)$$



Note: for 2nd and 3rd quadrants an angle adjustment is required. Hence,

$$\theta = tan^{-1}\left(\frac{\beta}{\alpha}\right) \pm 180°$$





CONTRIBUTOR

Stelios Ioannou