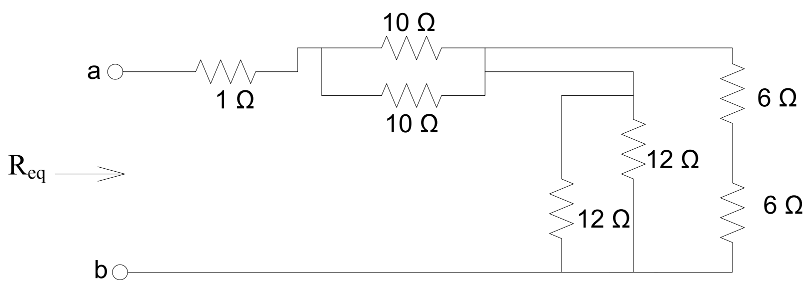
TOPIC

Electricity and Magnetism – Section XI – Question 8

QUESTION

The equivalent resistance in ohms of the circuit most nearly is



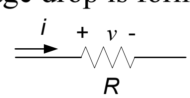
(A) 0.93
(B) 5.0
(C) 10
(D) 15

HINT

Resistors are electrical elements designed to exhibit resistance. This resistance opposes the direction of current by producing a voltage drop. From Ohm's law the voltage drop is proportional to the current

$$V = I\,R.$$

The symbol for a resistor is a *zig-zag* line. When there is a current flowing through a resistor then a voltage drop is formed



Resistance is represented with the capital letter R and it is measured in Ohms (Ω).

$$R = \frac{\rho A}{\ell}$$

where

ρ is the resistivity of the material (Ω.m),
A is the cross sectional area (m^2) and
ℓ is the length (m).

Resistors do not save electrical energy. Resistors absorb or dissipate energy as heat. As mentioned earlier power of any electrical element is the product of current and voltage;

$$P=IV$$

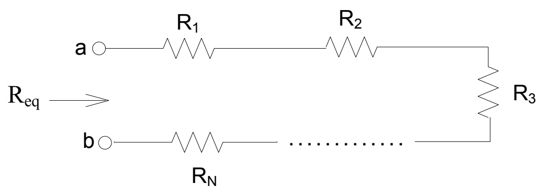
Only for resistors, power is also given by

$$P_R = i^2 R = \frac{v^2}{R},$$

where

i is the current through the resistor and
v is the voltage drop across it.

Two more useful terminologies to know are the short and open circuit.
A short circuit is when R = 0 Ω and voltage is zero for any given current.
An open circuit is when R = ∞ Ω and current is zero for any given voltage.
Resistors in series:



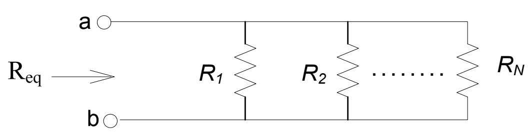
Resistors in series are added.

$$R_{eq} = R_1 + R_2 + R_3 + \ldots + R_N.$$

Note that a piece of wire (or short circuit) is a resistor with zero resistance ($R = 0\Omega$) and does not change the series resistance.

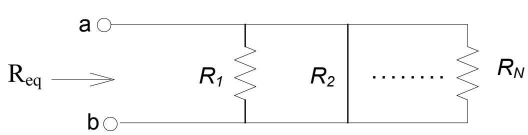
Resistors in Parallel: The notation is

$$R_{eq} = R_1 \,//\, R_2 \,//\ldots//\, R_N$$



$$R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots+\frac{1}{R_N}}$$

Note that a short circuit in parallel with a number of resistors connected in parallel is not insignificant as is the case with resistors connected in the series.

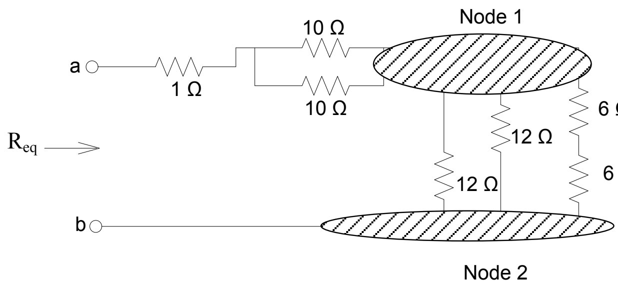


Even if one resistor becomes a short circuit ($R = 0\ \Omega$) then

$$R_{eq} = R_1 \,//\, 0 \,//\ldots//\, R_N = 0\ \Omega.$$

SOLUTION

We are interested for the equivalent resistance looking from left to right. This means that we will start from the right side and slowly work our way to the left. *Note that the resistance of a circuit changes when the angle or side of viewing the circuit changes*.



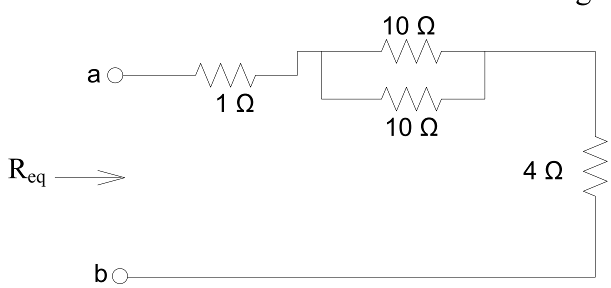
The two 6 Ω resistors are in series since they share 1 node and have the same current through them. Therefore,

$$6 + 6 = 12\ \Omega$$

Then the three 12 Ω resistors are in parallel since they share nodes 1 & 2. Therefore, $12//12//12 = \frac{1}{\frac{1}{12}+\frac{1}{12}+\frac{1}{12}}$

$$= 4\Omega$$

Now the circuit is minimized to the following:



Therefore, equivalent resistance is now clearly seen as 1Ω in series with (10//10) in series with the 4Ω.

$$R_{eq} = 1 + \frac{1}{\frac{1}{10}+\frac{1}{10}} + 4$$

$$= 10\ \Omega.$$

ANSWER

(C)

CONTRIBUTOR

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