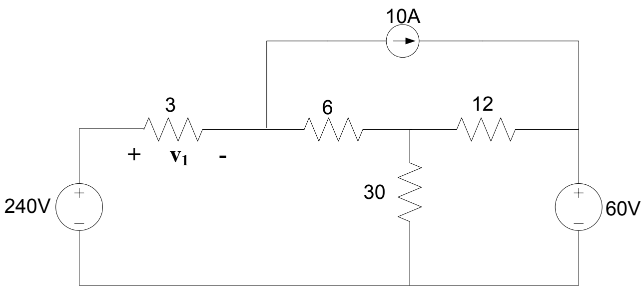
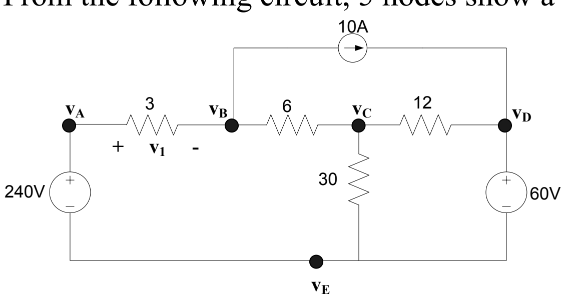
TOPIC
Electricity and Magnetism – Section XI – Question 9

QUESTION
The voltage v_1 in volts most nearly is



(A) 58.50
(B) 124.4
(C) 181.5
(D) 240.0

HINT
Kirchoff's current law (KCL) is a useful technique to calculate unknown currents. In addition KCL can be extended to what is known as *Nodal Analysis* to calculate nodal voltage in a circuit. There are three simple steps in performing nodal analysis;
1. Assign Nodal Voltages.
From the following circuit, 5 nodes show a system of 5 equations and 5 unknowns.



2. Pick a node and give it a known reference value. Keep things simple and make it a ground (zero volts).

SOLUTION
If we know a nodal voltage then we do not perform KCL on that node. Hence, by assigning v_E=0V then instantly we have minimized our system to 4 equations and 4 unknowns. Usually, the bottom node is chosen or a node of a voltage source.
The advantage of choosing the ground at one side of a voltage source is that we can minimize our work even more. A voltage source or any voltage drop have polarity denoted by + and -. The positive it is the high potential whereas the negative is the low potential.

To illustrate this look at the 240V source. High potential is v_A and low potential is v_E. Hence, it can be said that

$v_A - v_E = 240$.

Similarly, for the 60V source

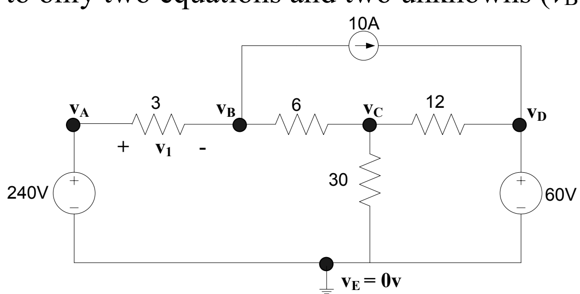
$v_D - v_E = 60$.

Also, the unknown voltage v_1

$v_A - v_B = v_1$.

Always remember that it is the high potential minus the low potential.

Hence, by choosing $v_E=0$, then $v_A=240$ and $v_D=60$. So, now the system is minimized even more to only two equations and two unknowns (v_B and v_C).



Sum the currents on every node. Remember, from Ohm's law that

$i = v / R$.

Assume that all currents are leaving the node unless if it is a current source. Currents entering a node are negative and currents leaving are positive.

KCL at v_B: $\frac{v_B - v_A}{3} + \frac{v_B - v_C}{6} + 10 = 0$

KCL at v_C: $\frac{v_C - v_B}{6} + \frac{v_C - v_E}{30} + \frac{v_C - v_D}{12} = 0$

Simplifying these two equations and substituting for $v_A=240$, $v_D=60$ and $v_E=0$ then:

$2v_B - 480 + v_B - v_C + 60 = 0$

gives

$3v_B - v_C = 420$ (1)

and

$10v_C - 10v_B + 2v_C - 0 + 5v_C - 300 = 0$

gives

$-10v_B + 17v_C = 300$ (2)

Solving equations (1) and (2), we get

$v_B = 181.5$ V and

$v_C = 124.4$ V

Then

$v_1 = v_A - v_B$

$= 240 - 181.5$

$= 58.5$V

ANSWER

(A)

CONTRIBUTOR

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