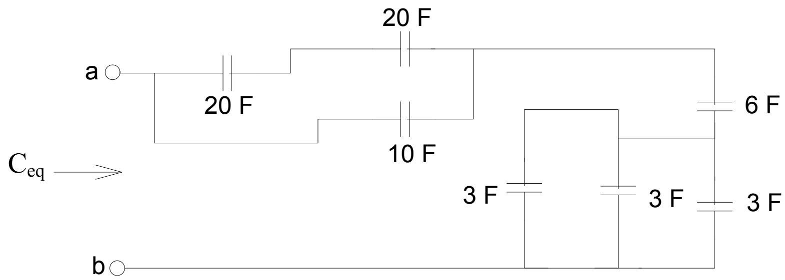
TOPIC

Electricity and Magnetism – Section XI – Question 11

QUESTION

The equivalent capacitance in Farads of the circuit below is most nearly



(A) 3.051
(B) 7.320
(C) 15.00
(D) 20.00

HINT

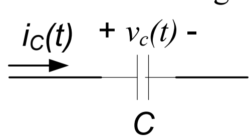
A capacitor is composed of two electrodes ('plates') separated by an electrolyte. The capacitance is directly proportional to electrode surface area, A in m^2 and inversely proportional to the separation distance between the electrodes, d in meters, and ε is the permittivity of the electrolyte.

$$C = \frac{A\varepsilon}{d}$$

Capacitors store energy (charge) in the form of an electric field; $Q = CV$, where Q is charge in Coulombs, C is capacitance in Farads and V is the voltage drop across the capacitor in volts. The total energy stored in a capacitor is given by:

$$E_C = \frac{1}{2}C{V_C}^2$$

Given the voltage drop across a capacitor then the current is the derivative of voltage. Hence, given the current then the voltage is the integral of current.



$$i_C(t) = C\frac{dV_C}{dt}$$

$$v_C(t) = v_C(0) + \frac{1}{C}\int_0^t i_C(\tau)d\tau$$

Looking closely to the current through a capacitor, some interesting properties can be concluded about capacitors.

For DC signals ($f = 0$Hz and $T = \infty$ seconds)

$$i_C(t) = C\frac{d}{dt}[const] = 0A$$

The capacitor acts like an open circuit. The current is known to be 0A but the voltage drop across the capacitor is unknown and needs to be calculated.

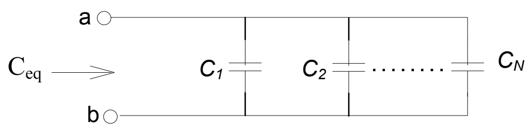
Instantaneous (*dt* = 0 seconds) changes in voltage drop across a capacitor

$$i_C(t) = C\frac{dv_C}{0} = \infty A$$

The voltage drop across a capacitor cannot be changed instantaneously (*dt* = 0 seconds) because that would cause an infinite current. Hence, it is said that $vc(0+) = vc(0-)$. Notice that the same cannot be said for the current through a capacitor.

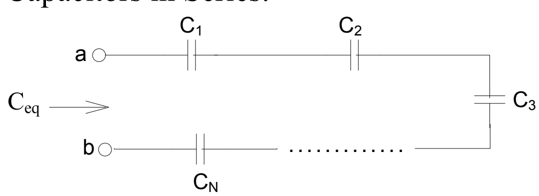
Equivalent Capacitance

Capacitors in Parallel:



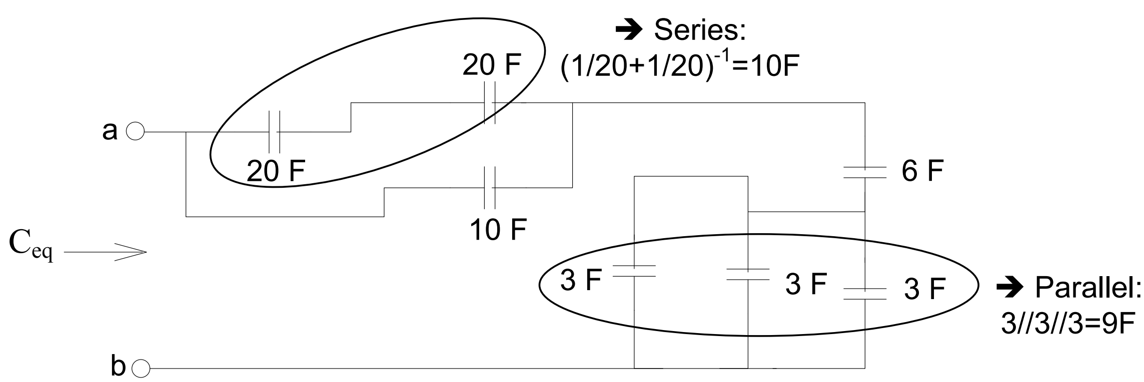
$C_{eq} = C_1 + C_2 + \cdots + C_N$

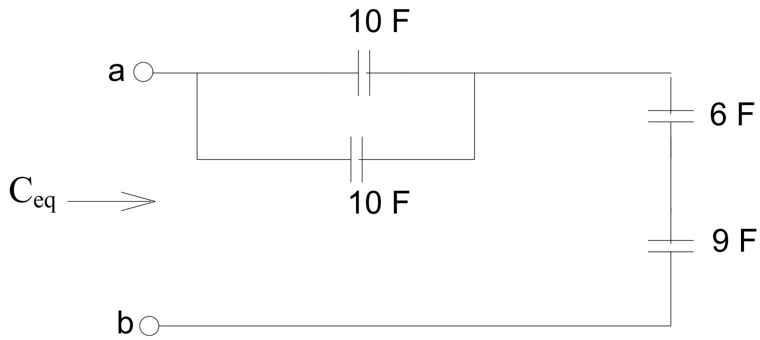
Capacitors in Series:



$$C_{EQ} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots+\frac{1}{C_N}}$$

SOLUTION





C_{eq} = (10//10) =10+10=20 in series to 6 in series to 9
= $(1/20+1/6+1/9)^{-1}$
= 3.051F

ANSWER
(A)

CONTRIBUTOR
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