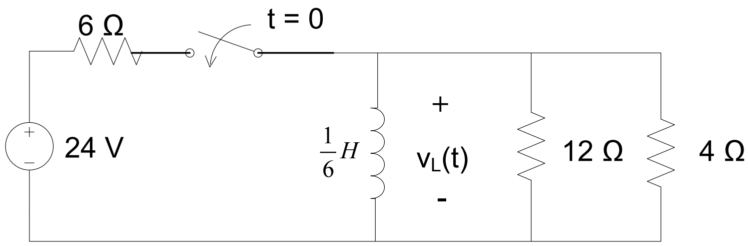
TOPIC

Electricity and Magnetism – Section XI – Question 15

QUESTION

The expression for the inductor voltage v_L(t) for $t > 0$ is



(A) $-12e^{-18t}$
(B) $-2e^{-3t}$
(C) $4-4e^{-12t}$
(D) $8e^{-12t}$

HINT

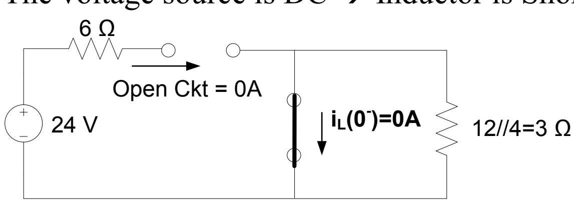
Also, note that the 12Ω and the 4Ω resistors are in parallel to the inductor. So, they have the same voltage drop across them. If the question was asking for the voltage drop across either of the two resistors then to keep things simple we would still solve for $i_L(t)$ then differentiate to get $v_L(t)$ and $v_4(t) = v_{12}(t) = v_L(t)$. Furthermore, the current through the 4Ω resistor would be $v_L(t)/4$ whereas the current through the 12Ω resistor would be $v_L(t)/12$.

SOLUTION

$t < 0$ Initial Conditions

Switch is open

The voltage source is DC → Inductor is Short Circuit → $v_L(0\text{-})$=0v → find $i_L(0\text{-})$



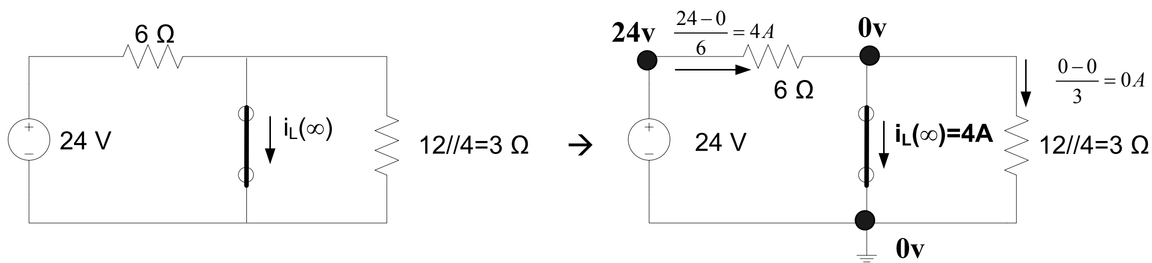
The current in the loop is zero because the switch is creating an open circuit in series to the source.

$i_L(0^-)$=0A. [No energy is supplied to the inductor]

$t = \infty$ Forced Response

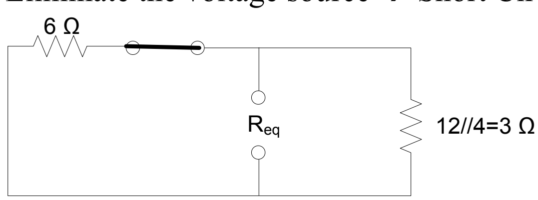
Switch was closed

The voltage source is DC → Inductor is Short Circuit → $v_L(\infty)$=0A → find $i_L(\infty)$



Since, the inductor is a short circuit then the top right node is 0v, the current through the 3Ω resistor is 0A, the current through the 6Ω is 4A and $i_L(\infty)$=4A.

$t > 0$
Find R_{eq}
Switch is closed
Eliminate the voltage source → Short Circuit



Across the terminals that the capacitor is connected what is the equivalent resistance as seen by the capacitor?
The 6 Ω is not floating this time.
Therefore, at t>0 the capacitor is looking at 6//3 Ω resistor.
R_{eq} = 6//3 = 2 Ω

Final Solution

$$i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{\frac{-tR_{eq}}{L}}$$
$$= 4 + [0 - 4]e^{\frac{-2t}{1/6}}$$
$$= 4 - 4e^{-12t}$$

Therefore,

$$v_L(t) = L\frac{di_L}{dt}$$
$$= \frac{1}{6}\frac{d}{dt}[4 - 4e^{-12t}]$$
$$= \frac{1}{6}(-4)(-12)$$
$$= 8e^{-12t}$$

$v_L(t) = 8e^{-12t}$ [Note that $v_L(0-)$ = 0V whereas $v_L(0^+)$= $8e^{-12(0)}$ = 8V].

ANSWER
(D)

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