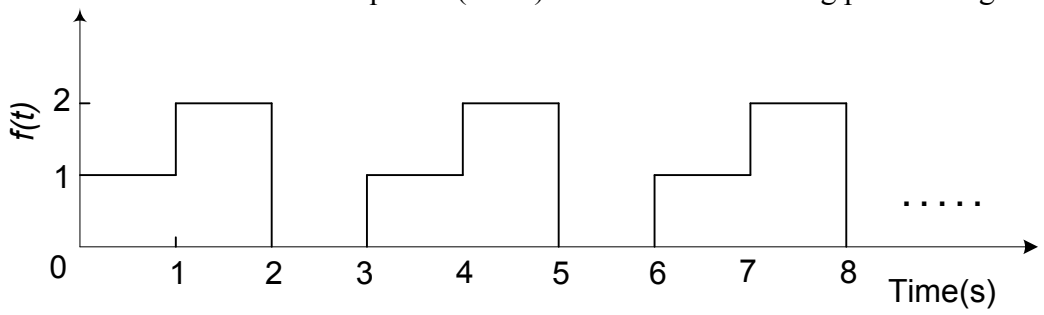
## TOPIC

Electricity and Magnetism – Section XI – Question 17

## QUESTION

The effective or root-mean squared (RMS) value of the following periodic signal most nearly is




(A) 0.701.
(B) 1.00
(C) 1.29.
(D) 2.00

## HINT

Average Value

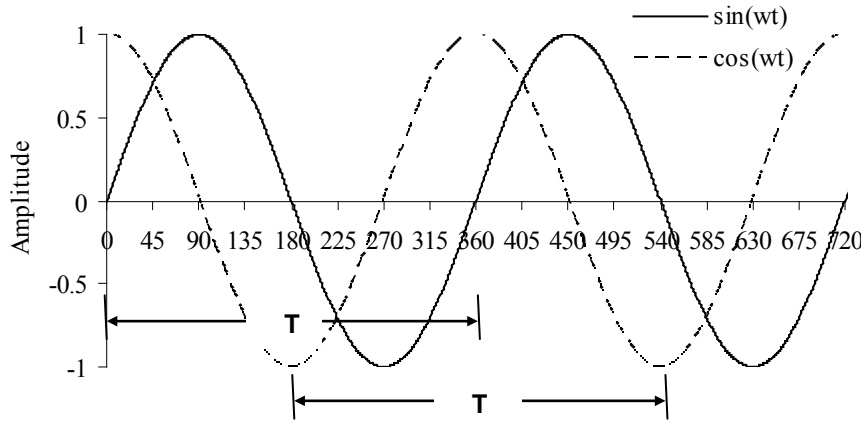
$$f_{av} = \frac{1}{T}\int_0^T f(t)dt$$

Effective or Root Mean Square (RMS)

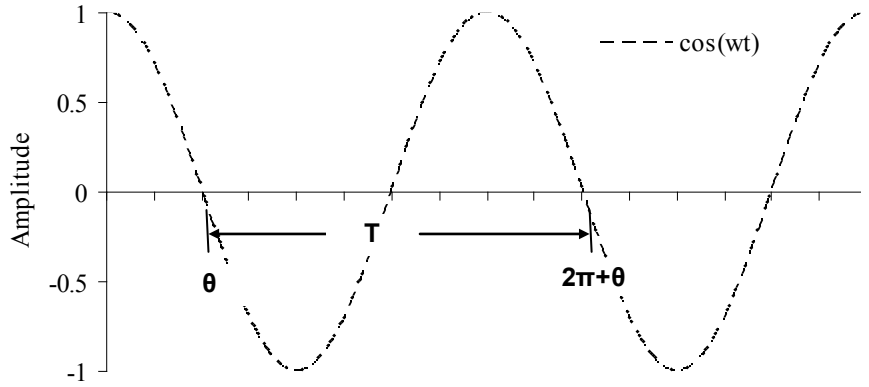
$$f_{RMS} = \sqrt{\left(\frac{1}{T}\int_0^T [f(t)]^2 dt\right)}$$

The period is defined as the interval taken for the signal to start repeating itself.

Example: Any Sinusoid




The period of any sinusoid independent of phase shift is $2\pi$ radians or $360^o$.
Average Value of any sinusoid independent of phase shift is zero, that is, $A\sin(\omega t + \theta)$




Period is from θ to (2π + θ).

$$\omega = \frac{2\pi}{T}$$

$$f_{av} = \frac{1}{(2\pi+\theta)-\theta}\int_{\theta}^{2\pi+\theta}[A\,sin(\,\omega.t+\theta)]dt$$

$$= \frac{1}{2\pi}\left[\frac{-A\,cos(\,\omega.t+\theta)}{\omega}\right]_{\theta}^{2\pi+\theta}$$

$$= \frac{1}{2\pi}\left[\frac{-A\,cos(\,\omega.t+\theta)}{\omega}\right]_{\theta}^{2\pi+\theta}$$

$$= \frac{1}{2\pi}\left[\frac{-A\,cos\left(\left(\frac{2\pi}{2\pi}\right).(2\pi+\theta)+\theta\right)}{\left(\frac{2\pi}{2\pi}\right)} + \frac{A\,cos\left(\left(\frac{2\pi}{2\pi}\right).(\theta)+\theta\right)}{\left(\frac{2\pi}{2\pi}\right)}\right]$$

$$= \frac{1}{2\pi}\left[\frac{-A\,cos\big((1).(2\pi+\theta)+\theta\big)}{(1)} + \frac{A\,cos\big((1).(\theta)+\theta\big)}{(1)}\right]$$

$$= -A\,cos(2\pi+2\theta) + A\,cos(2\theta)$$

$$= \frac{A}{2\pi}[-(cos(\,2\pi)\,cos(\,2\theta) - sin(\,2\pi)\,sin(\,2\theta)) + cos(2\theta)]$$

$$= \frac{A}{2\pi}[-cos(\,2\theta) + cos(2\theta)]$$

$$= 0$$

The RMS Value of any sinusoid independent of phase shift is

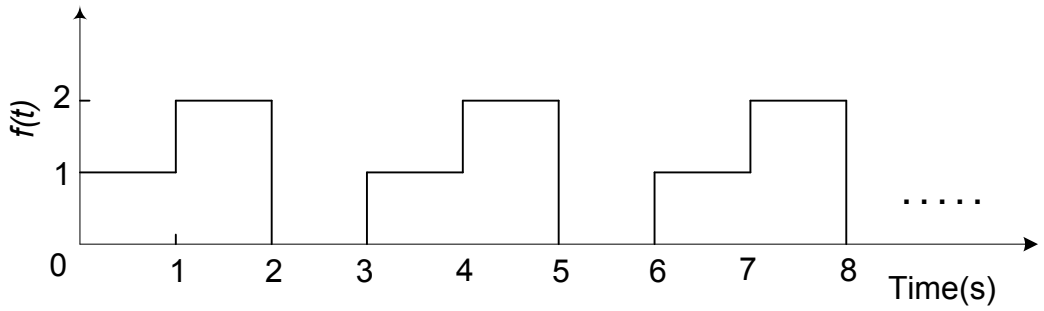
.

$$f_{RMS} = \sqrt{\frac{1}{(2\pi+\theta)-\theta}\int_{\theta}^{2\pi+\theta}[A\,sin(\,\omega.t+\theta)]^2dt}$$

$$= \sqrt{\frac{A^2}{2\pi}\int_{\theta}^{2\pi+\theta}[sin^2(\,\omega.t+\theta)]dt}$$

$$f_{RMS} = \frac{A}{\sqrt{2}}$$

**SOLUTION**




Period T = 3 seconds.

Average Value:

$$f_{av} = \frac{1}{3}\left[\int_{\theta}^{1} 1dt + \int_{1}^{2} 2dt + \int_{2}^{3} 0dt\right]$$
$$= \frac{1}{3}[t|_0^1 + 2t|_1^2 + 0|_2^3]$$
$$= \frac{1}{3}[1 + 2 + 0]$$
$$= 1$$

Effective or RMS Value

$$f_{RMS} = \sqrt{\frac{1}{3}\left[\int_{\theta}^{1} 1^2dt + \int_{1}^{2} 2^2dt + \int_{2}^{3} 0^2dt\right]}$$
$$= \sqrt{\frac{1}{3}[t|_0^1 + 4t|_1^2 + 0|_2^3]}$$
$$= \sqrt{\frac{1}{3}[1 + (8-4) + 0]}$$
$$= \sqrt{\frac{5}{3}}$$
$$=1.29$$

**ANSWER**

(C)

**CONTRIBUTOR**

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