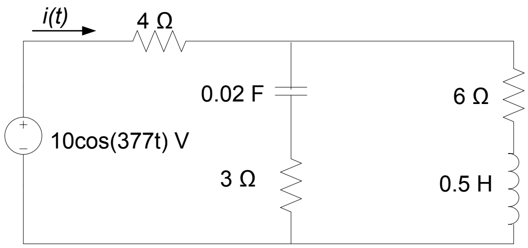
TOPIC

Electricity and Magnetism – Section XI – Question 18

QUESTION

The power factor (p.f.) for the following circuit most nearly is



(A) –0.9737
(B) 0.9737
(C) –13.18
(D) 13.18

HINT

So far we have learned that for DC signals (f = 0Hz), inductors are short circuit whereas capacitors are open circuit. Furthermore, inductors (in Henrys), capacitors (in Farads) and resistors in (Ohms) can not be combined. For example, a 1Ω resistor in series to a 1F capacitor cannot be added or manipulated in any way because the units are not the same. Similarly, we know that $I = V/R$ with units (Amperes = Volts/Ohms). Hence, $I \neq V/C$ because volts/Farads ≠ amperes and it is the same for inductors.

On the other hand, at the presence of an alternating current (AC) the resistors, inductor, and capacitors exhibit impedance in Ohms. Impedance is denoted with Z and has a magnitude and a direction.

$$\omega = 2\pi f$$

$\overline{Z}_R = R\angle 0° = R + j0$ [Resistor impedance is not frequency dependent]

$\overline{Z}_L = \omega L\angle 90° = 0 + j\omega L$ [Inductor impedance is directly proportional to f]

$\overline{Z}_C = \frac{1}{\omega C}\angle -90° = 0 - \frac{j}{\omega C}$ [Capacitor impedance is inversely proportional to f]

Does impedance violate anything we have learned so far about inductors and capacitors (i.e. f = 0Hz)? Let us check. f = 0Hz → ω=0 radians/second.

$\overline{Z}_R = R\angle 0° = R + j0$ [Resistor impedance is not frequency dependent]

$\overline{Z}_L = 0L\angle 90° = 0$ [Zero Ohms represents a short circuit]

$\overline{Z}_C = \frac{1}{0C}\angle -90° = \infty$ [Infinite Ohms represents an open circuit]

At any given frequency, once the capacitors and inductors are converted to impedances in Ohms then they can be treated like "resistors". When in series just add them, when in parallel then take the parallel combination and finally Ohm's law is now applicable $\overline{V} = \overline{I}.\overline{Z}$.

Resistors dissipate energy as heat, capacitors save energy in the form of an electric field whereas inductors save energy in the form of a magnetic field.

At the presence of an alternating current (AC) the resistors, inductor and capacitors exhibit impedance in Ohms. From Ohm's law $V=IZ$ hence Power = IV.
There are three kinds of AC power:
Real or Average power measured in Watts

$P = I_{RMS} V_{RMS} \cos(\theta_v - \theta_i)$.

Reactive Power measured in VARs

$Q = I_{RMS} V_{RMS} \sin(\theta_v - \theta_i)$

Complex Power measured in VA

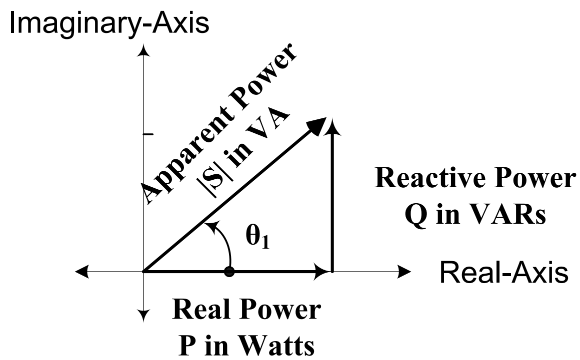
$S = V_{RMS} I^*_{RMS}$ [*I complex conjugate*]

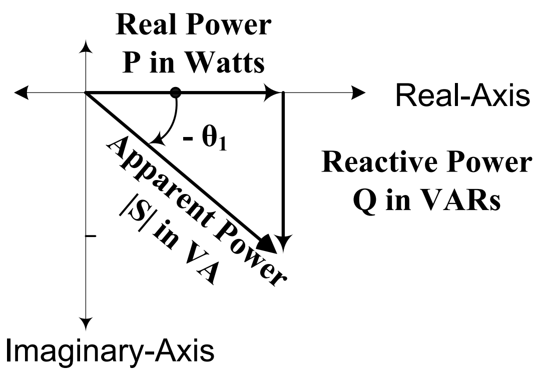
Together these three kinds of power form the *power triangle*, which can only be found in the first or fourth quadrants.
Complex Power is a vector:
In polar form:

$$\overline{S} = |\overline{S}|\angle \pm \theta°$$

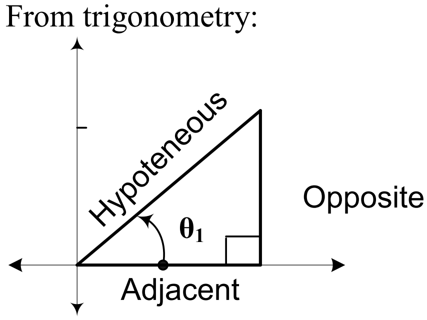
Apparent Power is |S|.
In rectangular form

$$\overline{S} = P \pm jQ$$





Power Factor (p.f.) equals $\cos(\theta)$.
Since, $\cos(\theta)=\cos(-\theta)$ then p.f. is always positive.
In first quadrant the p.f. is lagging.
In fourth quadrant the p.f. is leading.
From trigonometry:

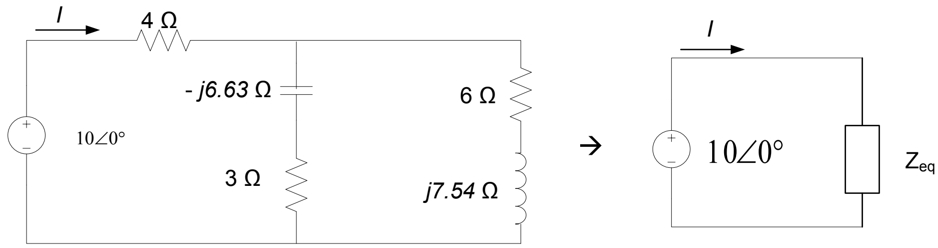


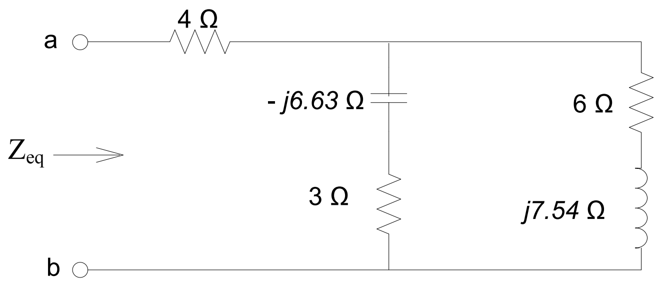
$\cos(\theta)$ = adjacent side / hypotenuse
$\sin(\theta)$ = opposite side / hypotenuse
$\tan(\theta)$ = opposite side / adjacent side

Therefore, since p.f.= cos(θ) then p.f.= $P / |S|$.

SOLUTION

Convert everything into polar form and impedances.
Trigonometric functions: Acos(ωt + θ) → Polar: $A\angle\theta°$
Hence, 10 cos(377t) → $10\angle 0°$
Also, ω=377 rads/sec.





The inductor impedance $\overline{Z}_L = 7.54\angle 90° = 0 + j7.54$ is in series to the 6 Ω

$$\overline{Z}_1 = \overline{Z}_L + 6 = 0 + j7.54 + 6 = 6 + j7.54 = 9.64\angle 51.5°$$

The capacitor impedance $\overline{Z}_C = 6.63\angle -90° = 0 - j6.63$ is in series to the 3Ω

$$\overline{Z}_2 = \overline{Z}_C + 3 = 0 - j6.63 + 3 = 3 - j6.63 = 7.28\angle -65.65°$$

Then impedances Z_1 and Z_2 are in parallel

$$\overline{Z}_3 = \overline{Z}_1 // \overline{Z}_2 = \left(\frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2}\right)^{-1}$$

$$\overline{Z}_3 = 7.287 - j2.644 = 7.752\angle -20°$$

Hence, equivalent impedance is the 4Ω in series to Z_3.

$$\overline{Z}_{eq} = 4 + \overline{Z}_3 = 4 + 7.287 - j2.644 = 11.287 - j2.644 = 11.593\angle -13.18°$$

Hence,

$$\overline{I} = \frac{\overline{V}}{\overline{Z}_{eq}} = \frac{10\angle 0°}{11.287 - j2.644} = \frac{10\angle 0°}{11.593\angle -13.18°} = 0.863\angle 13.18°$$

From polar to trigonometric form: The frequency does not change. This is why in polar form, the only quantities necessary are the magnitude and phase shift.

$$\overline{I} = 0.863\angle 13.18°$$

$i(t)$ = 0.863cos(377t + 13.18°)

$$\overline{S} = \left(\frac{10}{\sqrt{2}}\angle 0°\right)\left(\frac{0.863}{\sqrt{2}}\angle -13.18°\right)$$

$$= 4.315\angle -13.18°$$
$$= 4.2 - j0.984$$

p.f. = cos (–13.18)

= 0.9737

ANSWER

(B)

CONTRIBUTOR

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