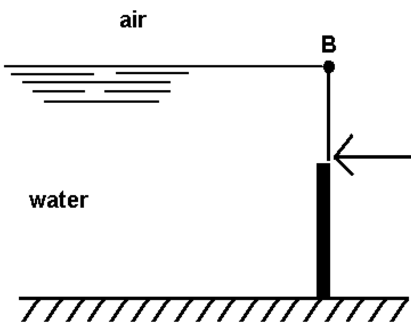
TOPIC
Fluids – Section X – Question 9

QUESTION
A gate is formed using a plate of dimension 1m ×1m hinged at point B as shown below. The force necessary to keep the gate from opening (in N) most nearly is

(A) 3280
(B) 4900
(C) 6570
(D) 9800



HINTS
- Find the force of the water on the plate
- Find the center of pressure through which this force acts
- Equate the moments of the force of the water and the force F around point B

SOLUTION
Since the pressure above the water surface and to the right of the gate is atmospheric at each location, its effect cancels and the pressure, P as a function of depth, is given by

$$P = \rho g y$$

where

y is the distance downward into the water from the surface,
ρ is the density, and
g is the acceleration due to gravity.

For a vertical gate, the force F_w of the water on the submerged gate is then given by

$$F_w = \rho g y_c A$$

where

A is the area of the gate, and
y_c is the centroid of the area.

Since the gate is 1 m in length, the centroid of the area will be at a depth of 0.5 m and the gate area is 1 m^2. The force of the water on the gate is thus

$$F_w = (9.8\text{m/s}^2)(1000\text{kg/m}^3)(0.5m)(1m^2) = 4900\text{kg-m/s}^2 = 4900N$$

However, the force of the water does not act through the centroid of the area but through the center of pressure. Since the pressure distribution on the gate is a triangle, the center of pressure will be one third the distance from the base of the triangle, or, in other words, at a depth of 0.67m. Alternatively, for a *vertical* gate, the distance from the centroid of the area y_c to the center of pressure y_p is given (see study manual) by

$$y_p = y_c + \frac{I}{Ay_c}$$

where I is the second moment of inertia of the gate around its centroidal axis. For a rectangular gate of width b and length L,

$$I = bL^3/12.$$

Thus

$$y_p = 0.5 + \frac{(1)(1)^3/12}{(1)(0.5)} = 0.67m$$

Equating the moments around the hinge at B

$$F_w y_p = FL$$

So

$$\begin{aligned} F &= F_w y_p/L \\ &= (4900N)(0.67m)/(1m) \\ &= 3283N \end{aligned}$$

ANSWER

(A)

CONTRIBUTOR

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