TOPIC
Mathematics – Section I – Question 1

QUESTION
The length of the curve $y = x^2$ from $x = 0$ to 3 most nearly is

(A) 3.000
(B) 9.000
(C) 9.487
(D) 9.747

HINT
The length of a curve $y(x)$ from $x = a$ to b is given by

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

SOLUTION
The length of a curve $y(x)$ from $x = a$ to b is given by

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$a = 0$

$b = 3$

$y = x^2$

$\frac{dy}{dx} = 2x$

$$s = \int_0^3 \sqrt{1 + (2x)^2}\, dx$$

$$= \int_0^3 \sqrt{1 + 4x^2}\, dx$$

$$= \left[\frac{x\sqrt{1+4x^2}}{2} + \frac{1}{4} sinh^{-1}(2x)\right]_0^3$$

$= 9.747$

ANSWER
(D)

CONTRIBUTOR
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