

**TOPIC**

Mathematics – Section I – Question 3

**QUESTION**

The Newton-Raphson method formula for finding the square root of a real number  $R$  from the equation  $x^2 = R$  is

(A)  $x_{i+1} = \frac{x_i}{2}$

(B)  $x_{i+1} = \frac{3x_i}{2}$

(C)  $x_{i+1} = \frac{1}{2} \left( x_i + \frac{R}{x_i} \right)$

(D)  $x_{i+1} = \frac{1}{2} \left( 3x_i - \frac{R}{x_i} \right)$

**HINT**

Rewrite the equation in the form  $f(x) = 0$ , that is,

$$f(x) = x^2 - R = 0$$

Now apply the Newton's formula

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

**SOLUTION**

The Newton's formula for finding the root of an equation is given by

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Since

$$x^2 = R$$

$$f(x) = x^2 - R = 0$$

$$f'(x) = 2x$$

$$\begin{aligned} x_{i+1} &= x_i - \frac{x_i^2 - R}{2x_i} \\ &= \frac{2x_i^2 - x_i^2 + R}{2x_i} \\ &= \frac{x_i^2 + R}{2x_i} \\ &= \frac{1}{2} \left( x_i + \frac{R}{x_i} \right) \end{aligned}$$

**ANSWER**

(C)

**CONTRIBUTOR**

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