TOPIC

Mathematics – Section I – Question 4

QUESTION

The form of the particular solution to the ordinary differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \sin x, \; y(0) = 5, \frac{dy}{dx}(0) = 6$$

is

(A) $Ax^2e^{2x} + B\sin x + C\cos x$
(B) $Ae^{2x} + B\sin x + C\cos x$
(C) $Ax^2e^{2x} + B\sin x$
(D) $Ae^{2x} + B\sin x$

HINT

The characteristic equation has repeated roots. The particular part of the solution will have the form of the right-hand side and its derivatives, unless they have the form of the homogeneous part of the solution.

SOLUTION

The characteristic equation of the ordinary differential equation is

$$m^2 - 4m + 4 = 0$$
$$(m-2)^2 = 0$$
$$m = +2, +2$$

So the homogeneous part of the solution is of the form

$$y_H = k_1e^{2x} + k_2xe^{2x}.$$

Corresponding to e^{2x}, the particular part would be

$$y_p = Ax^2e^{2x}$$

Corresponding to $\sin x$, the particular part of the solution would be

$$y_p = B\sin x + C\cos x.$$

Hence the full particular part of the solution is

$$y_p = Ax^2e^{2x} + B\sin x + C\cos x$$

ANSWER

(A)

CONTRIBUTOR

Autar Kaw