

TOPIC

Mathematics – Section I – Question 20

QUESTIONThe angle in degrees between two vectors \vec{u} and \vec{v}

$$\vec{u} = 3i + 5j + 7k$$

$$\vec{v} = 11i + 13j + 17k$$

most nearly is

(A) 8.124

(B) 11.47

(C) 78.52

(D) 81.88

HINT

$$\vec{u} \cdot \vec{v} = |u||v| \cos \theta$$

SOLUTIONThe dot product of two vectors $\vec{u} = (u_x, u_y, u_z)$ and $\vec{v} = (v_x, v_y, v_z)$ is

$$\begin{aligned}\vec{u} \cdot \vec{v} &= u_x v_x + u_y v_y + u_z v_z \\ &= 3 \times 11 + 5 \times 13 + 7 \times 17 \\ &= 219\end{aligned}$$

The dot product is also defined as

$$\vec{u} \cdot \vec{v} = |u||v| \cos \theta$$

$$\begin{aligned}u &= \sqrt{u_x^2 + u_y^2 + u_z^2} \\ &= \sqrt{3^2 + 5^2 + 7^2} \\ &= 9.11\end{aligned}$$

$$\begin{aligned}v &= \sqrt{v_x^2 + v_y^2 + v_z^2} \\ &= \sqrt{11^2 + 13^2 + 17^2} \\ &= 24.06\end{aligned}$$

$$\begin{aligned}\vec{u} \cdot \vec{v} &= |u||v| \cos \theta \\ &= 9.11 \times 24.06 \cos \theta \\ &= 219.2 \cos \theta\end{aligned}$$

Hence

$$219.2 \cos \theta = 217$$

$$\begin{aligned}\cos \theta &= \frac{217}{219.2} \\ &= 0.9900\end{aligned}$$

$$\theta = 8.124^\circ$$

ANSWER

(A)

CONTRIBUTOR

Autar Kaw