TOPIC

Engineering Probability and Statistics – Section II – Question 4

QUESTION

The yield of a chemical process is being studied. The past 5 days of plant operation have resulted in the following yields: 91.5, 88.7, 90.8, 89.9, and 92.1. Test hypotheses are H_0: mean yield μ = 90% versus H_1: $\mu \neq 90\%$. The P-value of this statistical test most nearly is

(A) 0.0500
(B) 0.2515
(C) 0.3125
(D) 0.4975

HINT

Since the variance of the yield is unknown, t distribution must be used. The P-value for a two-sided test is $2P(T_{n-1} > |t_0|)$, where n –1 are the degrees of freedom. Reject the null hypothesis H_0 at (1–α) significance level if α < P-value.

SOLUTION

The sample size is $n = 5$. Compute the sample average and sample standard deviation as $\bar{x} = 90.6$, and $s = 1.8$. Calculate

$$t_0 = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$$
$$= \frac{90.6-90}{1.8/\sqrt{5}}$$
$$= 0.745$$

$$\text{P-value} = 2P(T_4 > 0.745)$$
$$= 0.4975$$

ANSWER

(D)

CONTRIBUTOR

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