

TOPIC

Engineering Probability and Statistics – Section II – Question 8

QUESTION

The sodium content of a 300-gram box of organic corn flakes is approximately normally distributed with $\sigma = 0.55$ milligrams. Ten boxes were examined. The data (in milligrams) are as follows: 131.4, 130.9, 130.0, 129.8, 129.5, 129.4, 130.1, 130.6, 130.5, and 129.8. Test the hypothesis $H_0: \mu = 130$ versus $H_1: \mu \neq 130$, using $\alpha = 0.02$. If the true mean sodium content is 130.5 milligrams, the power of this test most nearly is

- (A) 0.050
- (B) 0.121
- (C) 0.216
- (D) 0.293

HINT

The power = $1 - \beta$, where β is the type-II error probability.

$\beta = P(\text{Accept } H_0: \mu = \mu_0, \text{ when } \mu = \mu')$.

SOLUTION

This is a z test, since σ is known.

$$z_{0.01} = 2.33$$

$$\mu_0 = 130$$

$$\mu' = 130.5$$

$$n = 10$$

Therefore,

$$\begin{aligned}\beta &= \Phi\left(z_{0.01} + \frac{(\mu_0 - \mu')\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{0.01} + \frac{(\mu_0 - \mu')\sqrt{n}}{\sigma}\right) \\ &= 0.293\end{aligned}$$

ANSWER

(D)

CONTRIBUTOR

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