TOPIC
Strength of Materials – Section VIII – Question 6

QUESTION
An engineer determines that a 100 KN force is required to punch a rectangular hole through a 5 mm thick steel plate with Young's modulus of 200 GPa and Poisson's ratio of 0.30. If the 200 mm long hardened steel punch is to make a 10mm by 30mm rectangular slot, the actual cross-sectional dimensions of the punch most nearly is (assume that no friction exists between the punch and plate)

(A) 10.0005 mm by 30.0015 mm
(B) 9.0005 mm by 29.0015 mm
(C) 9.995 mm by 29.985 mm
(D) 10 mm by 30 mm

HINT
The original cross sectional area of the punch would be smaller than the actual area of rectangular slot that is punched out. This is due to Poisson's effect. In other words, the cross sectional area increases due to the compressive axial stress.
Lateral strain is 5×10^{-4} m/m

SOLUTION
$$\varepsilon_{axial} = \frac{\sigma}{E}$$

$$\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows = \frac{P}{AE}$$

$$= \frac{-100000}{(0.010)(0.030)(200 \cdot 10^9)}$$
$$= -0.001667$$

$$\varepsilon_{lateral} = -\nu\varepsilon_{axial}$$
$$= -(0.30)(-0.001667)$$
$$= 0.0005$$

$$\Delta a = \varepsilon_{lateral}a$$
$$= (0.0005)(10)$$
$$= 0.005\text{mm}$$

$$\Delta b = \varepsilon_{lateral}b$$
$$= (0.0005)(30)$$
$$= 0.015\text{mm}$$

$$a_i = a_f - \Delta a$$
$$= 10 - 0.005$$
$$= 9.995\text{mm}$$

$$b_i = b_f - \Delta b$$
$$= 30 - 0.015$$
$$= 29.985\text{mm}$$

The actual cross-section of the punch is $a_i \times b_i = 9.995\text{mm} \times 29.985\text{mm}$

ANSWER
(C)

CONTRIBUTOR
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