TOPIC

Thermodynamics – Section XII – Question 6

QUESTION

Air is adiabatically compressed in a steady flow system from 1 bar to 5 bars. The air enters at 298 K and the compressor efficiency is 70 %. The work per unit mass of air required for the compression (in kJ/kg) most nearly is

(A) 122
(B) 174
(C) 249
(D) 1703

HINT

The steady state energy balance for this case is:

$$W/m = h_{in} - h_{out}$$

Where

W/m is work per unit mass and h is the enthalpy.

For an ideal gas with constant heat capacity (assumed here),

$$h_{in} - h_{out} = C_p(T_{in} - T_{out})$$

So that

$$W/m = C_p(T_{in} - T_{out})$$

SOLUTION

The outlet temperature is unknown. The procedure is to solve for W/m for the ideal case (where the outlet temperature can be calculated) and then apply the efficiency to get the actual W/m. The ideal case is reversible, and is also adiabatic. An adiabatic and reversible process is isentropic. For an isentropic process involving an ideal gas with constant heat capacity,

$$\frac{T_{\text{out,ideal}}}{T_{\text{in}}} = \left(\frac{P_{out}}{P_{in}}\right)^{\frac{k-1}{k}}$$

where T is the absolute temperature.

For air,

$k = 1.4$ and

$C_P = 1\text{kJ/kg-K}$

$$T_{\text{out,ideal}} = 298\left(\frac{5}{1}\right)^{\frac{0.4}{1.4}}$$
$$= 472K$$

Therefore

$$(W/m)_{ideal} = C_P(T_{in} - T_{out,ideal})$$

$$= 1(298 - 472)$$
$$= -174 \text{kJ/kg}$$

or 174 kJ are ideally required for each kg of air. The actual compressor requires more energy and is obtained from

$$W/m = (W/m)_{\text{ideal}}/\text{efficiency}$$
$$= 174/0.7$$
$$= 249 \text{kJ/kg}$$

The correct answer is (C). You get answer (A) if you multiply by the efficiency instead of dividing. Answer (B) is obtained if the efficiency is not used. Answer (D) is obtained if the incorrect (but commonly used) expression

$$\frac{T_{\text{out,ideal}}}{T_{\text{out}}} = \left(\frac{P_{out}}{P_{in}}\right)$$

is used.

ANSWER

(C)

CONTRIBUTOR

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